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# The Fatigue Strength of Members Containing Cracks

*The significant loss of fatigue life owing to the presence of a crack, crack-like defect, or sharp notch is predicted herein from theoretical considerations. Fatigue notch factors are given as functions of the crack depth, section width, and type of loading. These values are applicable where defects of a particular depth are known to exist or where defects of a limited depth could exist without being detected. Although these values apply specifically to 100,000 psi tensile strength steels, they are conservatively high for lower strength steels, aluminum, and other materials which are less sensitive to notches. The results of this paper indicate that cracks in finite-width members may produce a greater loss of fatigue life than previous theoretical work for members of infinite width had indicated.*

## Introduction

As far back as 1937, Neuber [1][1] pointed out that the theoretical stress concentration at the surface of a "pointed" notch obtained from the classical theory of elasticity is not representative of the peak stress in a real material.[2] Attempting to account for the effect of the "texture" of the material, he introduced the "material constant," $\rho'$, having the dimension of a length, and equal to half the width of an "elementary particle." Term $\rho'$ was designated by Neuber as the "elastic notch sensitivity," and was used as the minimum limiting value of the radius of curvature of the notch. Therefore in the transition to the pointed notch, the stress-concentration factor became a function of the material property $\rho'$.[3]

It is well known that peak local stress-concentration factors usually exaggerate the weakening effect of a stress concentration on the fatigue strength of a member. Part of this exaggeration is due to the ideal material assumption inherent in the theory of elasticity which Neuber attempted to take into account. This factor appears to the authors to be distinctly different from a second consideration suggested about the same time by R. E. Peterson [2] who pointed out that owing to the granular structure of the material, a finite volume of material must be stressed to the endurance limit in order to cause failure.[4] Peterson [3] and H. A. von Philipp [4] have since independently hypothesized that the stress at some finite distance $\delta$ below the surface is the stress value which limits fatigue life. The value of $\delta$ was assumed to be a property of the material, independent of the size of the specimen. This concept is generally referred to as the $\delta$-concept.

The use of the appropriate value of $\delta$ allows one to take account of both factors discussed previously. It also makes use of the observation[5] that fatigue fracture behaves as if the fatigue crack starts, not at the surface of the material, but at a small distance below the surface. The $\delta$-concept has been applied to various notch configurations [7, 8, 9] and reasonable agreement with fatigue-test results has been found.

Kuhn and Hardrath [10] proposed that Neuber's material property $\rho'$ be related to the tensile strength of steels by studying fatigue data from notched specimens and finding the $\rho'$-values

[1] Numbers in brackets designate References at end of paper.

[2] See especially pp. 162 and 163 of reference [1].

[3] Neuber devised an expression to take care of the transition from the actual notch curvature used in his mathematical expressions for weakly curved notches, and the minimum notch curvature used for sharp notches.

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[4] This concept is considered in a recent paper by R. Kuguel [5].

[5] According to Moore [6], this observation was made by R. E. Peterson during a meeting of the ASTM Research Committee on the Fatigue of Metals.

## Nomenclature

$A$ = constant used in equation (13) of Appendix and reference [1], lb/in.

$a$ = dimension shown in Figs. 2 and 4, in.

$c$ = distance to resultant tensile load shown in Fig. 4, in.

$E$ = modulus of elasticity, psi

$K$ = fatigue-strength reduction factor defined by equation (1), dimensionless

$K_T'$, $K_T''$ (tension), $K_B'$, $K_B''$ (bending) = fatigue notch factors for bars with deep notches on one side, and both sides, respectively. ($K_T'$ is for resultant load at center of net section), dimensionless

$K_i'$ = $K_T'$ or $K_B'$, respectively, dimensionless

$\overline{K_T}$, $\overline{K_B}$ = fatigue notch factors for a bar of arbitrary width containing a crack of arbitrary depth on one side, and subjected to cyclic tensile and bending loads, respectively ($\overline{K_T}$ is for resultant load at center or net section), dimensionless

$\overline{K_i}$ = $\overline{K_T}$ or $\overline{K_B}$, respectively, dimensionless

$\overline{\overline{K_T}}$ = fatigue notch factor for a bar of arbitrary width containing a crack of arbitrary depth on one side and subjected to cyclic tensile loads having a resultant at center of total section, dimensionless

$K_s$ = fatigue notch factors for bending and tension loads on bars with shallow cracks, dimensionless

$M$ = moment load per unit thickness, in-lb/in.

$M_1, M_2$ = moments designated in Fig. 5, in-lb/in.

$P$ = tensile load per unit thickness, lb/in.

$r$ = root radius of notch, in.

$t$ = depth of crack, in.

$T$ = width of section (see Fig. 12), in.

$r_0$ = notch contour parameter used in reference [1], dimensionless

$x, y$ = coordinates shown in Fig. 2, in.

$\delta$ = distance from surface of notch (see Fig. 2), in.

$\sigma_{nom}$ = nominal stress in net cross section, psi

$\sigma_\delta$ = $y$-direction stress at a distance $\delta$ from notch as shown in Fig. 2, psi

$\rho'$ = "elastic notch sensitivity" given by Neuber [1], in.

$\phi$ = dimensionless ratio defined by equation (5)



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